Indian Statistical Institute, Bangalore
B. Math (III)
Second Semester 2014-2015
Mid-Semester Examination : Statistics (V)
Sample Surveys and Design of Experiments.

Date: 06-03-2015 **Maximum Score 40** **Duration: 3 Hours**

1. Obtain π_i and π_{ij}, first and second order inclusion probabilities, $1 \le i \ne j \le N$, under *Midzuno-Sen sampling design.*

   [08]

2. Explain *Lahiri's method* and show that it indeed gives rise to *probability proportional to size (PPS) sampling.*

   $[4 + 6 = 10]$

3. The purpose of the survey is to estimate $\theta(w_1, w_2) = w_1\overline{Y}_1 + w_2\overline{Y}_2$, a linear combination of the stratum means $\overline{Y}_1$ and $\overline{Y}_2$, of two strata into which the population has been divided, w_1, w_2 are given real numbers. *Simple random sampling without replacement (SRSWOR)* samples of sizes n_1 and n_2 are to be selected from within strata independently. If the cost function is given by $C = c_1 n_1 + c_2 n_2$, find the best values of n_1 and n_2 for estimating θ. In particular consider the cases $a)$ $\theta = \overline{Y}_1 - \overline{Y}_2$, difference between the stratum means and $b)$ $\theta = \overline{Y}$, the population mean.

   [15]

4. Consider cluster sampling set up with N clusters. Let y_{ij} be the $y-$value of the j^{th} unit in the i^{th} cluster, $1 \le j \le M_i$ and $1 \le i \le N$, where M_i is the number of units in the i^{th} cluster, $1 \le i \le N$. Define $\overline{M} = \frac{1}{N}\sum_{i=1}^{N} M_i$. Let $\overline{Y}_i = \frac{1}{M_i}\sum_{j=1}^{M_i} y_{ij}$ be the mean of the i^{th} cluster, $1 \le i \le N$. Suppose we want to estimate the population mean $\overline{Y} = \frac{1}{N\overline{M}}\sum_{i=1}^{N}\sum_{j=1}^{M_i} y_{ij} = \frac{1}{N}\sum_{i=1}^{N}\frac{M_i\overline{Y}_i}{\overline{M}}$. Suggest an estimator for $\overline{Y}$, based on a sample of n clusters drawn using *simple random sampling with replacement (SRSWR).* Is your estimator unbiased? Obtain and estimate the *mean squared error (MSE)* of your estimator.

   $[2 + 3 + 3 + 4 = 12]$